Holographic inflation with tachyon field as an attractor solution*

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Abstract

We consider the attractor behaviour of the tachyon field dynamics in the holographic inflation cosmology. Our focus is on the model with exponential potential. The solutions of the dynamical equation in the phase-space are examined and we show they correspond to a dynamical attractor.

1. Introduction

Cosmological inflation is the leading theory for resolving some of the problems in the standard Big Bang model. Inflation predicts a short epoch during which the universe has suddenly expanded in size. The generated quantum fluctuations provided the seeds of cosmic microwave background anisotropy and the large scale structure [1].

One of the main features of the inflationary models, as well as one of the reasons to introduce inflation theory, is the assumption that the evolution of the universe, which can be described by a scalar field, is independent of the initial conditions. This property is regarded to the notion of cosmological

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attractors: dynamical conditions under which the evolving scalar fields approach a certain kind of behavior without fine tuning [2].

We analyze the properties of the attractor solutions in a model of inflation with tachyonic field in the framework of holographic cosmology. The use of the tachyon field in the inflation models is inspired by string theory [3]. We consider dynamics of the holographic model on a D3-brane located at the holographic boundary of an asymptotic ADS_5 bulk. Dynamics can be describe by effective four-dimensional Einstein equations on the holographic boundary [4].

2. Tachyon dynamics in the holographic braneworld

Under assumption that the holographic braneworld is spatially flat, the line element is of the form

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)(dr^2 + r^2 d\Omega^2). \tag{1}$$

In this case, following the procedure given in the ref. [5], the holographic Friedmann equations can be derived from effective four-dimensional Einstein equations

$$h^2 - \frac{1}{4}h^4 = \frac{\kappa^2}{3}\ell^4\rho, \tag{2}$$

$$\dot{h}\left(1 - \frac{1}{2}h^2\right) = -\frac{\kappa^2}{2}\ell^3(p+\rho), \tag{3}$$

where ℓ is the AdS curvature radius and $\kappa^2 = 8\pi G_{\mathrm{N}}/\ell^2$ is the fundamental dimensionless coupling. The overdot denotes derivative with respect to a dimensionless time variable $\tau = t/\ell$, while a dimensionless expansion rate is $h = \dot{a}/a$ [5]. The time evolution of the energy density ρ and the pressure p of the tachyon field θ is governed by the Lagrangian of the Dirac-Born-Infeld type [6]

$$\mathcal{L} = -\ell^{-4}V(\theta/\ell)\sqrt{1 - g^{\mu\nu}\theta_{,\mu}\theta_{,\nu}}\,, \tag{4}$$

i.e.

$$\rho = \frac{\ell^{-4}V}{\sqrt{1-\dot{\theta}^2}}, \quad p = -\ell^{-4}V\sqrt{1-\dot{\theta}^2}. \tag{5}$$

Here, V denotes a tachyon potential. The field θ is already redefined, so that its dimension is l. The energy conservation equation

$$\dot{\rho} + 3\frac{h}{l}(\rho + p) = 0, \tag{6}$$

gives a second order differential equation for $\theta(\tau)$

$$\frac{\ddot{\theta}}{1-\dot{\theta}^2} + 3\frac{h}{l}\dot{\theta} + \frac{V_{,\theta}}{V} = 0, \tag{7}$$

where $_{,\theta}$ denotes derivative with respect to theta. The equation (2) gives two solutions, but in the low density limit ($\kappa^2\ell^4\rho \ll 1$) only solution [5, 6]

$$h^2 = 2\left(1 - \sqrt{1 - \frac{\kappa^2}{3}\ell^4\rho}\right), \tag{8}$$

is reducible to the Friedmann equation in the standard cosmology. It follows that the physical range of the expansion rate is $0 \leq h^2 \leq 2$.

In the slow-role regime, when the field changes slowly under time due to the conditions [5]

$$\dot{\theta}^2 \ll 1, \quad |\ddot{\theta}| \ll 3\frac{h}{\ell}\dot{\theta}, \tag{9}$$

and while $\dot{\theta}$ can be approximated by

$$\dot{\theta} \simeq -\frac{\ell V_{,\theta}}{3hV}, \tag{10}$$

the equation (8) takes the form

$$h^2 \simeq 2\left(1 - \sqrt{1 - \frac{\kappa^2}{3}V}\right). \tag{11}$$

There are several ways to define the slow-roll parameters to ensure that the field changes slowly in time. We use the definition for the horizon-flow parameters ε_j given in the ref. [7]

$$\varepsilon_0 = \frac{h_*}{h}, \qquad \varepsilon_{i+1} = \frac{d\ln|\varepsilon_i|}{dN}, \quad i \geq 0, \tag{12}$$

where h_* is the Hubble rate at some chosen time and N is the number of e-folds. During inflation we have $\varepsilon_1 < 1$, $\varepsilon_2 < 1$ and inflation ends once either of the two parameters exceeds unity.

3. Attractor behavior - general consideration

To analyze the dynamics of the tachyon field it is useful to remind on the properties of an attractor behavior.

The definition of an attractor in the cosmological context is argued in ref. [2]. In cosmology, the concept of an attractor solution is introduced slightly different and less rigorous than in the mathematical literature [8]. Most often the term attractor is related to the specific properties of the depicted solutions in the phase space. Although the properties of the attractors depend on the choice of coordinates, the phase space we are going to work with will be represented as $(\theta, \dot{\theta})$ plane, as it is usually done.

Regarding to the ref. [9] the attractor solution can be found demanding

$$\frac{d\dot{\theta}}{d\theta} \simeq 0. \tag{13}$$

The equation (7) can be rewritten in the form

$$\frac{d\dot{\theta}}{d\theta} \simeq -\left(3\frac{h}{l}\dot{\theta} + \frac{V_{,\theta}}{V}\right)\frac{1-\dot{\theta}^2}{\dot{\theta}}, \tag{14}$$

where $\frac{d\dot{\theta}}{d\theta}$ is equal to zero if the expression in the parenthesis is equal to zero. This requirement leads to the approximate expression (10) and has been fulfilled only during the slow-roll regime. This is consistent with the well known fact that the slow-roll approximation provides an attractor solution ref. [10].

4. Attractor solutions

In a similar manner, as it was proposed in the ref. [11], we demonstrate the attractor behaviour of our model. We consider the expansion rate as a function of the inflaton field $h = h(\theta)$. Using equations (2) and (3), the time derivative of the tachyon field, in terms of the dimensionless expansion rate and its first derivative with respect to tachyon field, is given by

$$\dot{\theta} = -\frac{2}{3}\ell\frac{1-\frac{1}{2}h^2}{h^2-\frac{1}{4}h^4}h_{,\theta}\,, \tag{15}$$

This allows us to rewrite the equation (2) as the first order ordinary differential equations

$$9(h^2-\frac{1}{4}h^4)^2 - 4\ell^2(1-\frac{1}{2}h^2)^2 h_{,\theta}^2 = \kappa^4 V^2. \tag{16}$$

The equation (16) is of the Hamilton-Jacobi type equation. Suppose that $h_0(\theta)$ is its solution in the case of a general potential, which is able to support inflation. In order to examine the attractor behavior of θ, we consider the expansion rate h. Taking the perturbation $\delta h(\theta)$ of the solution $h_0(\theta)$ for the expansion rate

$$h(\theta) = h_0(\theta) + \delta h(\theta), \tag{17}$$

and substituting (17) in the equation (16), keeping only the terms up to the first order of the perturbation, we find

$$\frac{9}{4}h^3\frac{4-h^2}{2-h^2}\frac{d\theta}{h_{,\theta}} = \frac{d\delta h}{\delta h}. \tag{18}$$

Integrating the above expression, using the equation (15) to expres $h_{,\theta}$, and the equation (10) for $\dot{\theta}$, one gets

$$\delta h(\theta) = \delta h(\theta_i) e^{-3N}, \tag{19}$$

where the number of e-folds N is defined as in the reference [5]

$$N \equiv \frac{1}{\ell} \int_{t_i}^{t_f} h dt \simeq -3 \int_{\theta_i}^{\theta_f} \frac{h^2 V}{\ell^2 V_{,\theta}} d\theta. \tag{20}$$

The subscripts i and f are regarded to the beginning and to the end of inflation, respectively. To solve the horizon and the flatness problem, we need about 50-60 e-folds. The perturbation δh decreases as we approach the end of inflation, and the model possesses the attractor behavior. Therefore, the all solutions quickly approach each other and they are independent of initial conditions, as we demonstrate in the following section.

5. Attractor behavior with exponential potential

We consider the following exponential potential

$$V = e^{-\omega|\theta|/\ell}, \tag{21}$$

where ω is a dimensionless parameter. This potential is one of the most frequently used for the tachyon systems [12]. The value of the parameter ω is determined by the initial value of expansion rate h_i and the number of e-folds N. From the equation (20), and using the condition $\varepsilon_{2f} = 1$ for the end of inflation, one gets an approximate expression [5]

$$\omega^2 \simeq \frac{12\left[\frac{h_i^2}{2} + \ln\left(1 - \frac{h_i^2}{4}\right)\right]}{N+1}. \tag{22}$$

The initial value of the field can be estimated from (11), giving [5]

$$\theta_i \simeq -\frac{1}{\omega} \ln\left[\frac{3}{\kappa^2}\left(1 - (1 - \frac{h_i^2}{2})^2\right)\right]. \tag{23}$$

To examine behavior of the inflationary solution in the phase-space we need to find the solution of the equation (14), which for the exponential potential takes the form

$$\frac{d\dot{\theta}}{d\theta} \simeq -\frac{1}{l}\left(3\frac{h}{l}\dot{\theta} - \omega\right)\frac{1-\dot{\theta}^2}{\dot{\theta}}. \tag{24}$$

The equation (24) is solved numerically for a variety of arbitrary chosen initial values. Using equations (10) and (11) one can find the relation between θ and $\dot{\theta}$ which is valid in the slow-roll regime

$$\dot{\theta} \simeq \frac{\omega}{\sqrt{18(1-\sqrt{1-\frac{\kappa^2}{3}e^{-\omega|\theta|/\ell}})}}. \tag{25}$$

As it is shown in the section 3., slow roll approximation is necessary in order to provide a solution that is an attractor solution. Although the equation (14) is independent of κ, we need to choose a numerical value for κ in order to set the initial value for $\dot{\theta}$, and we choose $\kappa = 1$.

In our model inflation ends when $\varepsilon_{2f} = 1$, and the inflaton field reaches the value [5]

$$\theta_f = \frac{\ell}{\omega} \ln \frac{\kappa^2}{\omega^2}. \tag{26}$$

The obtained diagram, in $(\theta, \dot{\theta})$ plane, is presented in the (Fig. 1).

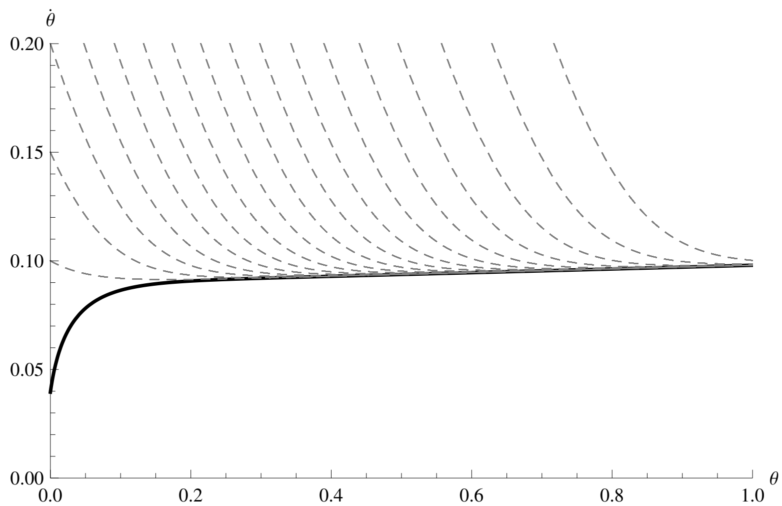

Figure 1: $\dot{\theta}$ versus θ diagram, calculeted numerically for several initial values of $\dot{\theta}$ and fixed parameter $\omega = 0.164$. The solid line represents the solution obtained in respect to the initial values constrained by the equation (25).

For many different choices of the initial conditions the phase space trajectories converge at late time and there exists a curve that attracts most of the trajectories. According to the criteria given in the reference [2] the solution in the model behaves as an attractor.

6. Conclusions

In this paper, we consider the inflation model with a tachyon field in holographic cosmology and it is shown that the solutions for the tachyon field θ have properties of the an attractor. It is shown that the use of the Hamilton-Jacobi equation (16) is very effective in this consideration. Also, in the case of the exponential potential a numerical solution of the dynamical equation is found for a wide interval of the initial values and plotted in a the phase space $(\theta, \dot{\theta})$. The numerical results fully confirm the existence of the attractor behaviour.

It is an interesting task to examine and demonstrate the attractor behaviour for other tachyon-type potentials, and to check if any possible violation of the attractor behaviour may appear.

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