Cosmological solutions of a nonlocal square root gravity

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Motivation

Large cosmological observational findings:

- High orbital speeds of galaxies in clusters. (F.Zwicky, 1933)
- High orbital speeds of stars in spiral galaxies. (Vera Rubin, at the end of 1960es)
- Accelerated expansion of the Universe. (1998)

Problem solving approaches

There are two problem solving approaches:

- Dark matter and energy
- Modification of Einstein theory of gravity

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}, \; c = 1$$

where $T_{\mu\nu}$ is stress-energy tensor, $g_{\mu\nu}$ are the elements of the metric tensor, $R_{\mu\nu}$ is Ricci tensor and R is scalar curvature of metric.

Dark matter and energy

- If Einstein theory of gravity can be applied to the whole Universe then the Universe contains about 4.9% of ordinary matter, 26.8% of dark matter and 68.3% of dark energy.
- It means that 95.1% of total matter, or energy, represents dark side of the Universe, which nature is unknown.
- Dark matter is responsible for orbital speeds in galaxies, and dark energy is responsible for accelerated expansion of the Universe.

Modification of Einstein theory of gravity

Motivation for modification of Einstein theory of gravity

- The validity of General Relativity on cosmological scale is not confirmed.
- Dark matter and dark energy are not yet detected in the laboratory experiments.
- Another cosmological problem is related to the Big Bang singularity. Namely, under rather general conditions, general relativity yields cosmological solutions with zero size of the universe at its beginning, what means an infinite matter density.
- Note that when physical theory contains singularity, it is not valid in the vicinity of singularity and must be appropriately modified.

Approaches to modification of Einstein theory of gravity

There are different approaches to modification of Einstein theory of gravity.

- Einstein General Theory of Relativity

From action $S = \int (\frac{R}{16\pi G} - L_m - 2\Lambda)\sqrt{-g}d^4x$ using variational methods we get field equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}, \; c = 1$$

where $T_{\mu\nu}$ is stress-energy tensor, $g_{\mu\nu}$ are the elements of the metric tensor, $R_{\mu\nu}$ is Ricci tensor and R is scalar curvature of metric.

Nonlocal Modified Gravity

Nonlocal gravity is a modification of Einstein general relativity in such way that Einstein-Hilbert action contains a function f(□, R). Our action is given by

$$S = \frac{1}{16\pi G} \int \left(R - 2\Lambda + \sqrt{R - 2\Lambda}\mathcal{F}(\Box)\sqrt{R - 2\Lambda} \right) \sqrt{-g}\, d^4x$$

where $\Box = \frac{1}{\sqrt{-g}} \partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu$, $\mathcal{F}(\Box) = \sum_{n=1}^{\infty} f_n \Box^n$.

We use Friedmann-Lemaître-Robertson-Walker (FLRW) metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right), \ k \in \{-1, 0, 1\}.$$

Equations of motion

Equation of motion are

$$-\frac{1}{2}g_{\mu\nu}\sqrt{R-2\Lambda}\mathcal{F}(\Box)\sqrt{R-2\Lambda}+R_{\mu\nu}W-K_{\mu\nu}W+\frac{1}{2}\Omega_{\mu\nu}=-\frac{G_{\mu\nu}+\Lambda g_{\mu\nu}}{16\pi G},$$

$$\Omega_{\mu\nu}=\sum_{n=1}^{\infty}f_n\sum_{l=0}^{n-1}S_{\mu\nu}(\Box^l\sqrt{R-2\Lambda},\Box^{n-1-l}\sqrt{R-2\Lambda}),$$

$$K_{\mu\nu}=\nabla_\mu\nabla_\nu-g_{\mu\nu}\Box,$$

$$S_{\mu\nu}(A,B)=g_{\mu\nu}\nabla^\alpha A\nabla_\alpha B-2\nabla_\mu A\nabla_\nu B+g_{\mu\nu}A\Box B,$$

$$W=\frac{1}{\sqrt{R-2\Lambda}}\mathcal{F}(\Box)\sqrt{R-2\Lambda}.$$

Trace and 00-equations

In case of *FRW* metric there are two linearly independent equations. The most convenient choice is trace and 00 equations:

$$-2\sqrt{R-2\Lambda}\mathcal{F}(\Box)\sqrt{R-2\Lambda}+RW+3\Box W+\frac{1}{2}\Omega=\frac{R-4\Lambda}{16\pi G},$$
$$\frac{1}{2}\sqrt{R-2\Lambda}\mathcal{F}(\Box)\sqrt{R-2\Lambda}+R_{00}W-K_{00}W+\frac{1}{2}\Omega_{00}=-\frac{G_{00}-\Lambda}{16\pi G},$$
$$\Omega=g^{\mu\nu}\Omega_{\mu\nu}.$$

The ansatz

At first we analyze the ansatz $\Box(R+R_0)^m = p(R+R_0)^m$, $m, p, R_0 \in \mathbb{R}$.
Scale factor is in the form

$$a(t) = A\, t^n e^{-\frac{\gamma}{12}t^2}.$$

Ansatz in the expanded for is

$$
\begin{aligned}
&-648mn^2(2n-1)^2(2m-3n+1)=0,\\
&-324n(2n-1)\left(-\gamma m+6\gamma mn^2-4\gamma mn-mnR_0+mR_0+2n^2r-nr\right)=0,\\
&18n(2n-1)\left(8\gamma^2m^2-13\gamma^2m+12\gamma^2mn-3\gamma mR_0+24\gamma nr+6\gamma r-6rR_0\right)=0\\
&-2\gamma^3m-24\gamma^3mn^2-14\gamma^3mn+6\gamma^2mnR_0+2\gamma^2mR_0+72\gamma^2n^2r+12\gamma^2nr\\
&-24\gamma nrR_0+3\gamma^2r-6\gamma rR_0+3rR_0^2=0,\\
&-\gamma^2\left(4\gamma^2m^2+\gamma^2m+18\gamma^2mn-3\gamma mR_0-24\gamma nr-6\gamma r+6rR_0\right)=0,\\
&-\gamma^4(r-\gamma m)=0.
\end{aligned}
$$

There are five solutions of the above system

1. $p=m\gamma,\ n=0,\ R_0=\gamma,\ m=\frac{1}{2}$
2. $p=m\gamma,\ n=0,\ R_0=\frac{\gamma}{3},\ m=1$
3. $p=m\gamma,\ n=\frac{1}{2},\ R_0=\frac{4}{3}\gamma,\ m=1$
4. $p=m\gamma,\ n=\frac{1}{2},\ R_0=3\gamma,\ m=-\frac{1}{4}$
5. $p=m\gamma,\ n=\frac{2}{3},\ R_0=\frac{7}{3}\gamma,\ m=\frac{1}{2}.$

Case $n = \frac{2}{3}$, $m = \frac{1}{2}$

Trace and 00 equations split into the following systems:

$$\mathcal{F}'(\frac{\gamma}{2}) = 0, \qquad \frac{11\gamma}{3} + 4\Lambda - \gamma\mathcal{F}(\frac{\gamma}{2}) = 0,$$
$$1 + \mathcal{F}(\frac{\gamma}{2}) = 0, \qquad \gamma^2 + \gamma^2\mathcal{F}(\frac{\gamma}{2}) = 0,$$

and

$$\mathcal{F}'(\frac{\gamma}{2}) = 0, \qquad -\frac{2}{3}\gamma - \Lambda + \frac{1}{2}\gamma\mathcal{F}(\frac{\gamma}{2}) = 0,$$
$$1 + \mathcal{F}(\frac{\gamma}{2}) = 0, \qquad \gamma^2 + \gamma^2\mathcal{F}(\frac{\gamma}{2}) = 0.$$

The solution is

$$\mathcal{F}(\frac{\gamma}{2}) = -1, \qquad \mathcal{F}'(\frac{\gamma}{2}) = 0, \qquad \gamma = -\frac{6}{7}\Lambda.$$

- We take the scale factor in the form $a(t) = At^n \exp(-\frac{\gamma}{12}t^2)$.
- Ansatz is in the form $\Box(R+R_0)^m = r(R+R_0)^m$.
- Ansatz has two solutions, for $m = \frac{1}{2}$.
- In case $n = 0$, $m = \frac{1}{2}$ there is unique solution $\mathcal{F}(\frac{\gamma}{2}) = -1$, $\mathcal{F}'(\frac{\gamma}{2}) = 0$ where $\gamma = \Lambda$.
- In case $n = \frac{2}{3}$, $m = \frac{1}{2}$ there is unique solution $\mathcal{F}(\frac{\gamma}{2}) = -1$, $\mathcal{F}'(\frac{\gamma}{2}) = 0$ where $\Lambda = -\frac{7}{6}\gamma$.

- We take the scale factor in the form $a(t) = A\exp(\lambda t)$, $k \neq 0$.
- This scale factor satisfies the ansatz $\Box\sqrt{R-2\Lambda} = \frac{\Lambda}{3}\sqrt{R-2\Lambda}$, for $\Lambda = 6\lambda^2$
- EOM are satisfied for $\mathcal{F}(\frac{\Lambda}{3}) = -1$, $\mathcal{F}'(\frac{\Lambda}{3}) = 0$.
- There are two solutions for $\lambda = \pm\sqrt{\frac{\Lambda}{6}}$.

- We look for solutions with constant scalar curvature R.
- From EOM we obtain that $R = 4\Lambda$.

Depending on the sign of Λ we have the following scale factors. If Λ is positive then

- $a(t) = Ae^{\pm\sqrt{\frac{\Lambda}{3}}t}, \ k = 0,$
- $a(t) = \sqrt{\frac{3}{\Lambda}}\cosh\sqrt{\frac{\Lambda}{3}}t, \ k = 1,$
- $a(t) = \sqrt{\frac{3}{\Lambda}}|\sinh\sqrt{\frac{\Lambda}{3}}t|, \ k = -1.$

On the other hand if Λ is negative then there is one solution $a(t) = \sqrt{-\frac{3}{\Lambda}}|\cos\sqrt{-\frac{\Lambda}{3}}t|$ for $k = -1$.

Effective $T_{\mu\nu}$

Let us consider the following two solutions

$$a_1(t) = At^{\frac{2}{3}} e^{\frac{\Lambda}{14} t^2},$$
$$a_2(t) = Ae^{\frac{\Lambda}{6} t^2}.$$

The EOM can be rewritten in the form

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G \bar{T}_{\mu\nu}.$$

Corresponding effective density and pressure are

$$\bar{\rho}_1 = \frac{\Lambda}{12\pi G}\left(2(\Lambda t^2)^{-1} + \frac{9}{98}\Lambda t^2 - \frac{9}{14}\right), \bar{p}_1 = -\frac{\Lambda}{56\pi G}\left(\frac{3}{7}\Lambda t^2 - 1\right).$$

$$\bar{\rho}_2 = \frac{\Lambda}{8\pi G}\left(\frac{1}{3}\Lambda t^2 - 1\right), \bar{p}_2 = -\frac{\Lambda}{24\pi G}\left(\Lambda t^2 - 1\right).$$

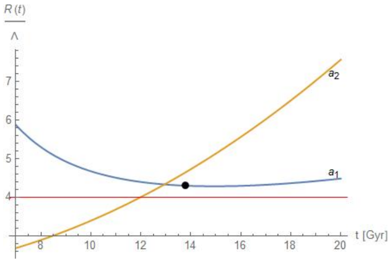

Figure: Scalar curvatures for scale factors a_1, a_2

Scalar curvatures for these solutions are given by

$$R_1 = \frac{4}{3}t^{-2} + \frac{22}{7}\Lambda + \frac{12}{49}\Lambda^2 t^2,$$
$$R_2 = 2\Lambda + \frac{4}{3}\Lambda^2 t^2.$$

Planck data

- Hubble parameter $H_0 = 67.40 km/s/Mpc$,
- Age of the universe $t_0 = 13.801 \cdot 10^9 years$,
- Matter density parameter $\Omega_m = 0.315$,
- Λ density parameter $\Omega_\Lambda = 0.685$,
- ratio of pressure to energy density $w_0 = -1.03$.

Using Planck values for H_0 and t_0 and

$$H(t) = \frac{2}{3t} + \frac{1}{7}\Lambda t, \tag{1}$$

we get

$$\Lambda = 1.05 \cdot 10^{-35} s^{-2}. \tag{2}$$

Also, one gets that the minimum of the function $H(t)$ is $H_m = 61.72 km/s/Mpc$ at $t_m = 21.1 \cdot 10^9 years$.

Similarly, from

$$\frac{a''(t)}{a(t)} = -\frac{2}{9}t^{-2} + \frac{\Lambda}{3} + \frac{\Lambda^2 t^2}{49}, \tag{3}$$

we get that accelerated expansion of the Universe started at $t_a = 7.84 \cdot 10^9 years$ or in other words $5.96 \cdot 10^9 years$ ago.

Let ρ_c be critical density and $\bar{\rho} = \bar{\rho}(t_0)$. Then

$$\bar{\Omega} = 0.265. \tag{4}$$

For the visible matter we take $\Omega_v = 0.05$ and then

$$\bar{\Omega}_\Lambda = 1 - \bar{\Omega} - \Omega_v = 0.685. \tag{5}$$

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Thank you!