Analytic Infinite Derivative (AID) field theories

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Based on recent papers in collaboration with L.Buoninfante, B.Dragovich, S.Korumilli, J.Marto, A.Mazumdar, L.Modesto, P.Moniz, L.Rachwal, A.Starobinsky, and others and the current works in progress

Instead of introduction

- **Einstein's gravity is not renormalizable**
- **Stelle's 1977 and 1978 papers show that R^2 gravity is renormalizable gravity with the price of a physical (Weyl) ghost**
- **Recall: Ostrogradski statement from 1850 forbids higher derivatives in general. The Weyl tensor already has 2, its square has 4 and constraints do not alleviate the problem.**
- **Good thing: Starobinsky inflation is based on R^2 and works perfectly**

The early Universe formation, which is most likely inflation, is for the time being perhaps the only testbed for testing gravity modifications.

So what?

We start with

$$S = \int d^D x \sqrt{-g} \left(\mathcal{P}_0 + \sum_i \mathcal{P}_i \prod_I (\hat{\mathcal{O}}_{iI} \mathcal{Q}_{iI}) \right)$$

Here $\mathcal{P}$ and $\mathcal{Q}$ depend on curvatures and $\mathcal{O}$ are operators made of covariant derivatives.

Everywhere the respective dependence is *analytic*.

The most general action to consider

We are looking for the most general action capturing in full generality the properties of a linearized model around *maximally symmetric space-times (MSS)* such that

$$R_{\mu\nu\alpha\beta} = f(x)(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})$$

The result is [arxiv.1602.08475]

$$S = \int d^D x\sqrt{-g}\Bigg(\frac{M_P^2 R}{2} - \Lambda$$

$$+\frac{\lambda}{2}\Big(R\mathcal{F}_R(\Box)R + L_{\mu\nu}\mathcal{F}_L(\Box)L^{\mu\nu} + W_{\mu\nu\lambda\sigma}\mathcal{F}_W(\Box)W^{\mu\nu\lambda\sigma}\Big)\Bigg)$$

Here $\mathcal{F}_X(\Box) = \sum_{n\geq 0} f_{X_n}\Box^n$ and $L_{\mu\nu} = R_{\mu\nu} - \frac{1}{D}Rg_{\mu\nu}$

Thanks to the Bianchi identities one can further achieve $\mathcal{F}_L(\Box) = 0$ in $D = 4$ and $\mathcal{F}_L(\Box) = \text{const}$ in $D > 4$.

Quadratic action around (A)dS with $\bar{R} = 4\Lambda/M_P^2$

The covariant decomposition is

$$h_{\mu\nu} = \frac{2}{M_P^2} h^{\perp}_{\mu\nu} + \bar{\nabla}_\mu A_\nu + \bar{\nabla}_\nu A_\mu + \left(\bar{\nabla}_\mu \bar{\nabla}_\nu - \frac{1}{4} \frac{2}{M_P^2} \sqrt{\frac{8}{3}} \bar{g}_{\mu\nu} \bar{\Box} \right) B + \frac{1}{4} \frac{2}{M_P^2} \sqrt{\frac{8}{3}} \bar{g}_{\mu\nu} h$$

Here $\bar{\nabla}^\mu h^{\perp}_{\mu\nu} = \bar{g}^{\mu\nu} h^{\perp}_{\mu\nu} = \bar{\nabla}^\mu A_\mu = 0.$

Vector part and $\bar{\nabla}_\mu \bar{\nabla}_\nu B$ **terms go away around MSS.**

Spin-2:

$$S_2 = \frac{1}{2}\int dx^4 \sqrt{-\bar{g}}\, h^{\perp}_{\nu\mu} \left(\bar{\Box} - \frac{\bar{R}}{6}\right) \left[\mathcal{P}(\bar{\Box})\right] h^{\perp\,\mu\nu}$$

$$\mathcal{P}(\bar{\Box}) = 1 + \frac{2}{M_P^2}\lambda f_{R_0}\bar{R} + \frac{2}{M_P^2}\lambda \mathcal{F}_W \left(\bar{\Box} + \frac{\bar{R}}{3}\right)\left(\bar{\Box} - \frac{\bar{R}}{3}\right)$$

The Stelle's case corresponds to $\mathcal{F}_W = 1$ such that

$$\mathcal{P}(\bar{\Box})_{Stelle} = 1 + \frac{2}{M_P^2}\lambda f_{R_0}\bar{R} + \frac{2}{M_P^2}\lambda \left(\bar{\Box} - \frac{\bar{R}}{3}\right)$$

This is an obvious second pole which will be the ghost.

Spin-0 (here $\phi \equiv \bar{\Box} B - h$):

$$S_0 = -\frac{1}{2}\int dx^4 \sqrt{-\bar{g}}\; \phi(3\bar{\Box} + \bar{R})\left[\mathcal{S}(\bar{\Box})\right]\phi$$

$$\mathcal{S}(\bar{\Box}) = 1 + \frac{2}{M_P^2}\lambda f_{R_0}\bar{R} - \frac{2}{M_P^2}\lambda \mathcal{F}_R(\bar{\Box})(3\bar{\Box} + \bar{R})$$

This *is* the ghost in Einstein-Hilbert case $\mathcal{F}_R = 0$, but it is constrained and is not physical.

Thus, $\mathcal{S}(\bar{\Box})$ *can* have one root to generate one pole and it will be not a ghost.

This would be exactly the scalar mode in a local $f(R)$ gravity.

Physical excitations

Effectively we modify the propagators as follows

$$\Box - m^2 \to \mathcal{G}(\Box)$$

Recall, in $D = 4$ in $(- + ++)$

$$L = \phi(\Box - m^2)\phi - \text{good field}$$

$-\Box$ gives a ghost, $+m^2$ gives a tachyon.

To preserve the physics we demand

$$\mathcal{G}(\Box) = (\Box - m^2)e^{\sigma(\Box)}$$

where $\sigma(\Box)$ must be an *entire* function resulting that the exponent of it has no roots.

We arrange this in our model by virtue of functions $\mathcal{F}$.

Entire functions

- **A function is *analytic* in some domain if it is expandable in it in the Taylor series**
- **A function is *entire* if it is analytic in the whole complex plane. The simplest are polynomials.**
- **An entire function is constant if it is analytic at infinity**
- **An exponent of an entire function is again an entire function but *without zeroes* in the complex plane**
- **If a function has a pole at infinity, its Taylor series at zero in $w = 1/z$ must have finite number of terms**
- **An exponent of en entire function would have an infinite Taylor series at zero in $w = 1/z$ and this corresponds to the *essential singularity***
- **At the point of the essential singularity the limit of a function depends on the direction in the complex plane.**

UV completeness

Minkowski propagator:

$$\Pi = -\left(\frac{P^{(2)}}{k^2 e^{H_2(-k^2)}} - \frac{P^{(0)}}{2k^2 e^{H_0(-k^2)} \left(1 + \frac{k^2}{M^2}\right)} \right)$$

To guarantee that the QFT machinery works we arrange a polynomial decay of the propagator near infinity. The rate of the decay is our choice.

Recall that we still need the functions $H_{0,2}$ to be entire.

An example of such a function can be, for instance

$$H \sim \Gamma\left(0, p(z)^2\right) + \gamma_E + \log\left(p(z)^2\right)$$

where $p(z)$ is a polynomial.

Beyond 1-loop the powercounting arguments work just like in the higher derivative regularization.

Amplitudes and Cross-sections

Power-counting works because we have chosen the polynomial decay at infinity

Slavnov-Taylor identities work thanks to the presence of the diffeomorphism invariance

Exponential decay of form-factors renders the system to be in the strong-coupling regime. This way amplitudes become divergent for large external momenta.

The ongoing work in progress with A.Tokareva aims to determine conditions on form-factors wich would retain standardly expected behavior of amplitudes.

Conclusions

- **A class of analytic infinite derivative (AID) theories has been considered**
- **A UV complete and unitary gravity is discussed**
- **It features many nice properties, like native embedding of the Starobinsky inflation, finite Newtonian potential at the origin, presence of a non-singular bounce, etc.**
- **The theory predicts a modified value for r for example**
- **The theory has clear connection to SFT**

Open questions

- More concrete understandning of how form-factors are constrained from the point of view of QFT

- Explicit demonstration of the absence of singular solutions in this model

- Deeper study of inflation and bouncing scenarios in this model

- Derive the graviton action from the SFT in the full rigor

Thank you for listening!