Thermodynamic Study of Reissner–Nordström Quintessence Black Hole

by

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MPHYS10

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Overview

What a Black Hole is

Highly dense collapsed massive star from which gravity prevents anything including light, from *escaping*.

- In Newtonian physics
  - the escape velocity($v_{esc} = \sqrt{2GM/R}$); if v_{esc} exceeds the speed of light then the corresponding object becomes BH
- In general relativity
  - Black hole is a region wrapped by event horizon
  - Example :*Schwarzchild Metric*
    $ds^2 = -(1 - \frac{2GM}{c^2 r})dt^2 + \frac{1}{(1-\frac{2GM}{c^2 r})}dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2)$
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Reissner-Nordström Black Hole

A static spherically symmetric solution for Einstein's field equations, surrounded by quintessence, can be expressed as

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2),$$

where, $$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{c}{r^{(3\omega_q+1)}} \quad .$$

The range of quintessential state parameter is $-1 < \omega_q < -\frac{1}{3}$.

$$\rho_q = -\frac{c}{2}\frac{3\omega_q}{r^{3(1+\omega_q)}}$$

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Thermodynamics

In order to find the black hole mass, we set $f(r) = 0$, which yields

$$M = \frac{r_+}{2}\left[1 + \frac{Q^2}{r_+^2} - \frac{c}{r_+^{(3\omega_q+1)}}\right] .$$

The entropy will take the form $S = \pi r^2$.

The electrostatic potential difference can be expressed as

$$\Phi = \left(\frac{\partial M}{\partial Q}\right)_S = \left(\frac{\partial M}{\partial Q}\right)_{r_+} = \frac{Q}{r_+} .$$

The Hawking temperature of the black hole is given by,

$$T_H = \frac{f'(r)}{4\pi}\bigg|_{r=r_+} = \frac{1}{4\pi}\left[\frac{1}{r_+} - \frac{Q^2}{r_+^3} + \frac{3c\omega_q}{r_+^{(3\omega_q+2)}}\right] .$$

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Now the specific heat will take the form,

$$C_Q = T\left(\frac{\partial S}{\partial T}\right)_Q = \frac{2\pi r_+^2\left(r_+^2 - Q^2 + 3\omega_q c r_+^{-(3\omega_q-1)}\right)}{3Q^2 - r_+^2 - 3\omega_q(3\omega_q+2)c r_+^{-(3\omega_q-1)}} .$$

In order to find a divergence in specific heat one must satisfy the condition, $3Q^2 - r_+^2 - 3\omega_q(3\omega_q+2)cr_+^{-(3\omega_q-1)} = 0$.

Fig.1a

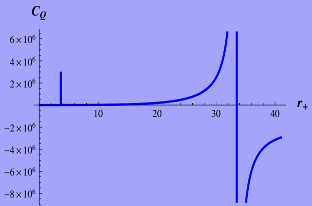

Fig.1b

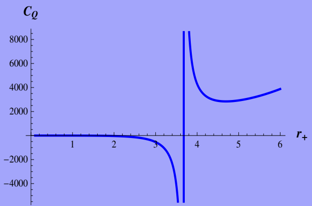

Fig: $(1a)$ and $(1b)$ represent the variation of specific heat with respect to r_+ for $\omega_q = -\frac{2}{3}$, $Q = 2$ and $c = 0.8$.

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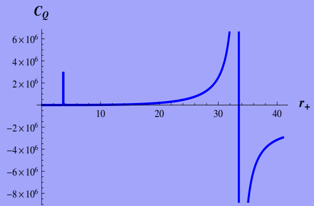

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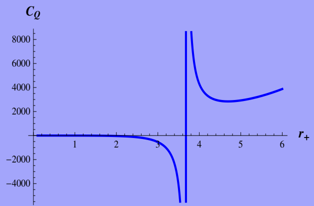

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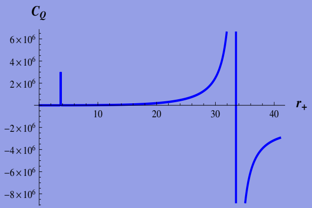

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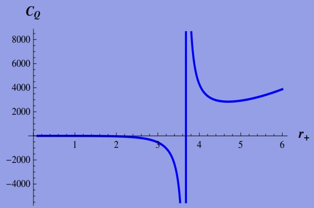

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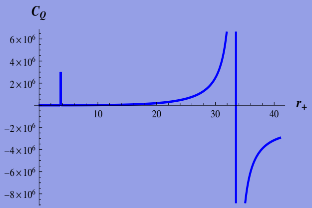

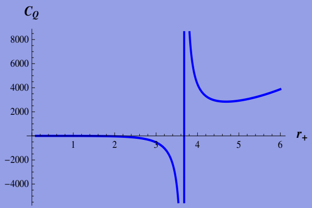

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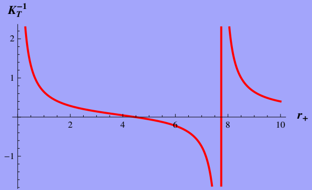
Thermodynamics

The inverse of the isothermal compresibility is given by,

$$K_T^{-1} = Q\left(\frac{\partial\phi}{\partial Q}\right)_T = \frac{Q}{r_+}\left(\frac{Q^2 - r_+^2 - 3\omega_q(3\omega_q+2)cr_+^{-(3\omega_q-1)}}{3Q^2 - r_+^2 - 3\omega_q(3\omega_q+2)cr_+^{-(3\omega_q-1)}}\right) .$$

Fig.2a



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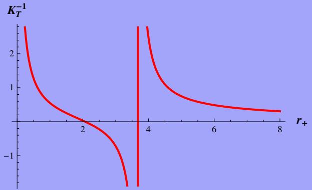

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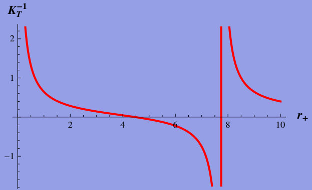

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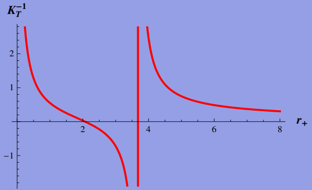

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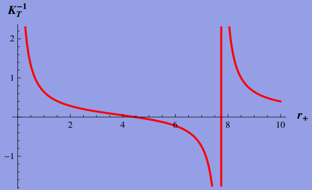

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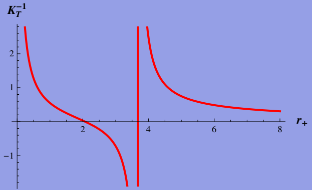

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Critical exponents

Over the past few decades, the study of critical phenomena has come to concentrate more on the values of a set of indices $(\alpha, \beta, \gamma, \delta, \varphi, \psi, \nu, \eta)$, known as critical exponents which play an important role to describe the singular behavior of various thermodynamic quantities near the critical points.

The standard definition of the critical exponents are,

$$C_Q \sim |T - T_c|^{-\alpha},$$
$$K_T^{-1} \sim |T - T_c|^{-\gamma},$$
$$\Phi(r) - \Phi(r_c) \sim |T - T_c|^{\beta},$$
$$\Phi(r) - \Phi(r_c) \sim |Q - Q_c|^{\frac{1}{\delta}},$$
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\Phi(r) - \Phi(r_c) \sim |Q - Q_c|^{\frac{1}{\delta}}, \\
C_Q \sim |Q - Q_c|^{-\varphi}, \\
S(r) - S(r_c) \sim |Q - Q_c|^{\psi} \; .
\end{gathered}$$

We would like to re-express physical quantities near the critical point as

$$r = r_c(1+\Delta),$$
$$T(r) = T(r_c)(1+\epsilon),$$
$$Q(r) = Q(r_c)(1+\Pi) \ ,$$

where, $\Delta << 1$, $\epsilon << 1$ and $\Pi << 1$.

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where, $\Delta << 1$, $\epsilon << 1$ and $\Pi << 1$.

Performing Taylor expansion of $T(r_+)$ for a fixed value of the charge in the neighborhood of r_c, we obtain

$$T = T(r_{+c}) + \left[\left(\frac{\partial T}{\partial r_+}\right)_{Q=Q_c}\right]_{r_+=r_{+c}} (r_+ - r_{+c}) + \frac{1}{2}\left[\left(\frac{\partial^2 T}{\partial r_+^2}\right)_{Q=Q_c}\right]_{r_+=r_{+c}} (r_+ - r_{+c})^2 + \text{higher order terms} .$$

Now, neglecting the higher order terms,

$$\epsilon T_c = \frac{1}{2}\left[\left(\frac{\partial^2 T}{\partial r^2}\right)_Q\right]_{r_+=r_c} r_c^2 \triangle^2$$

Using the re-expressed quantities we get,

$$\triangle = \frac{1}{r_c}\sqrt{\frac{2\epsilon T_c}{D}}, \quad \text{where,} \quad D = \left[\left(\frac{\partial^2 T}{\partial r^2}\right)_{Q=Q_c}\right]_{r_+=r_c} .$$

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Now, we can expanding the denominator of C_Q near the critical point as

$$C_Q = \frac{2\pi r_+^2 (r_+^2 - Q^2 + 3\omega_q c r_+^{-(3\omega_q - 1)})}{\triangle \left[-2r_+^2 + 3\omega_q (3\omega_q - 1)(3\omega_q + 2) r_+^{-(3\omega_q - 1)} \right]} ,$$

which can be transformed into,

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Next, we evaluate the critical exponent β which is associated with the electrostatic potential (Φ) for a fixed value of charge Q and defined through the relation as

$$\Phi(r_+) - \Phi(r_c) \sim |T - T_c|^\beta \ .$$

Let us first, we use the Taylor expansion of $\Phi(r_+)$ in the neighborhood of r_c,

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Scaling Law

Critical exponents	α	β	γ	δ	φ	ψ
Values	1/2	1/2	1/2	2	1/2	1/2

Now we discuss about the *thermodynamic scaling law* for our present work. These relations(laws) are stated below:

$$\alpha + 2\beta + \gamma = 2,$$
$$\alpha + \beta(\delta + 1) = 2,$$
$$(2-\alpha)(\delta\psi - 1) + 1 = (1-\alpha)\delta,$$
$$\gamma(\delta + 1) = (2-\alpha)(\delta - 1),$$
$$\beta\delta = \beta + \gamma,$$
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Discussions

Based on standard thermodynamic approach, we have provided a general scheme which could be employed to study the critical phenomena in R-N black holes. Using a canonical ensemble, we have exploited this scheme to study the critical behavior in Reissner-Nordström Black Holes surrounded by quintessence.

The obligatory sketch out of charged BH thermodynamics with the central engine amplified into a quintessence field is
Stable small BH → Second order phase transition → Unstable small/intermediate mass BH → First order phase transition → Stable intermediate mass BH → Second order phase transition → unstable super massive BHs.

Based on this novel approach we have calculated all the static critical exponents which satisfy the so called thermodynamic scaling relations near the critical point.

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THANKS