Memory effect of massive gravitational waves

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Outline

- Introduction (What is memory effect)
- Massive gravitational waves in 3D and their memory
- Massive gravitational waves in 4D and their memory

Introduction

- We have a system of test masses in asymptotically flat spacetime
- Passage of a gravitational wave induces observable disturbance on a system of test masses
  Zel'dovich and Polnarev
- Displacement memory effect
  Zel'dovich and Polnarev, Thorn, Christodoulou
- Velocity memory effect
  Braginsky and Grishchuk, Grishchuk and Polnarev, Bondi and Pirani, Zhang et al
- Connected with BMS symmetry at null infinity
  Strominger et al

Introduction

Geodesic equations

$$\frac{d^2x^\mu}{d^2\lambda} + \Gamma^\mu_{\nu\rho}\frac{dx^\nu}{d\lambda}\frac{dx^\rho}{d\lambda} = 0\,. \tag{1}$$

Asymptotically

$$\Gamma^\mu_{\nu\rho} \to 0 \; when \; \lambda \to \infty\,. \tag{2}$$

Consequently, the asymptotic solution of geodesic equations is

$$x^\mu = a^\mu\lambda + b^\mu\,. \tag{3}$$

There are three possible scenarios

- $a^\mu = F(initial\ conditions)$ Velocity memory effect
- $a^\mu = const,\ b^\mu = F(initial\ conditions)$ Displacement memory effect
- $a^\mu = const,\ b^\mu = const$ No memory

Massive gravitational waves in 3D

Theory under consideration is Poincare gauge theory of gravity which is mostly quadratic in curvature and torsion and parity invariant

$$L = -*a_0 R + T^i \sum_{n=1}^{3} *a_n T_i^{(n)} + \frac{1}{2} R^{ij} \sum_{n=4}^{6} *b_n R_{ij}^{(n)}\,, \qquad (4)$$

M. Blagojević and B. Cvetković, Phys. Rev. D**90** (2014).

Massive gravitational waves in 3D

Ansatz for the metric is

$$ds^2 = H(u,y)du^2 + 2dudv - dy^2\,, \tag{5}$$

from which we obtain vielbein

$$e^+ = du\,,\ \ e^- = \frac{1}{2}Hdu + dv\,,\ \ e^2 = dy\,. \tag{6}$$

Ansatz for spin connection

$$\omega^{ij} = \tilde{\omega}^{ij} + \frac{1}{2}\varepsilon^{ij}{}_m k^m k_n e^n K(u,y)\,, \tag{7}$$

where $k^n = (1,-1,0)$. The solution in spin 2 sector with tordion mass m is given by

$$H(u,y) = A(u)\cos my + B(u)\sin my\,, \tag{8}$$

$$H \propto \partial_y K\,. \tag{9}$$

Geodesic equations

Non-zero Riemann connections

$$\Gamma^{v}{}_{uu} = \frac{1}{2}\partial_u H\,,\ \ \Gamma^{y}{}_{uu} = \frac{1}{2}\partial_y H\,,\ \ \Gamma^{v}{}_{uy} = \frac{1}{2}\partial_y H\,. \tag{10}$$

Geodesic equation for u is

$$\frac{d^2u}{d^2\lambda} = 0\,, \tag{11}$$

so we can chose $\lambda = u$.The rest of geodesic equations are

$$\frac{d^2y}{d^2u} + \frac{1}{2}\partial_y H = 0\,, \tag{12}$$

$$\frac{d^2v}{d^2u} + \frac{1}{2}\partial_u H + \partial_y H\frac{dy}{du} = 0\,. \tag{13}$$

Velocity memory effect $\frac{1}{2}H = -\frac{1}{u}\cos y$

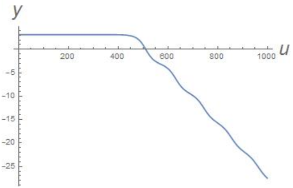
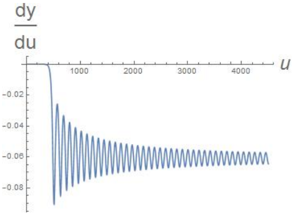


Figure: Graphics y[u], Initial conditions $y[1] = \pi$, $y'[1] = 0$.



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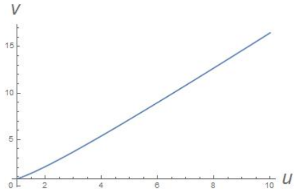


Figure: Graphics v[u], Initial conditions $v[1] = \pi/4\,,\; v'[1] = 0\,.$

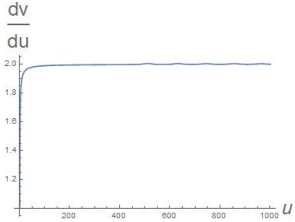


Figure: Graphics dv[u]/du, Initial conditions $v[1] = \pi/4\,,\; v'[1] = 0\,.$

Velocity memory effect $\frac{1}{2}H = -e^{-(u-10)^2}\cos y$

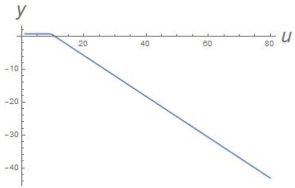
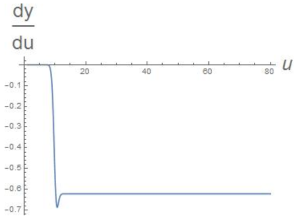


Figure: Graphics y[u], Initial conditions $y[0] = \pi/4\,,\ y'[0] = 0\,.$



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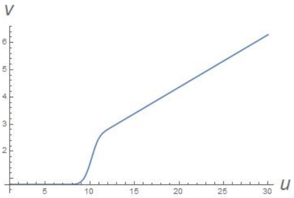

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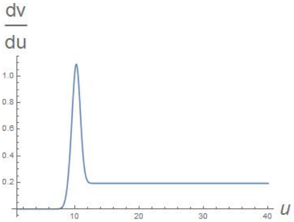

Figure: Graphics dv[u]/du, Initial conditions $v[0] = \pi/4\,,\ v'[0] = 0\,.$

Massive gravitational waves in 4D

We consider Poincare gauge theory which Lagrangian is at most quadratic in curvature and torsion and parity invariant. Lagrangian 4-form is given by

$$L = - * a_0 R + T^i \sum_{n=1}^{3} * a_n T_i^{(n)} + \frac{1}{2} R^{ij} \sum_{n=1}^{6} * b_n R_{ij}^{(n)} \, .$$

M. Blagojević and B. Cvetković, Phys. Rev. D**95** (2017).
M. Blagojević, B. Cvetković and Y. N. Obukhov, Phys. Rev. D**96** (2017)

Massive gravitational waves in 4D

Ansatz for the metric is direct generalization of 3D case

$$ds^2 = H(u, y, z)du^2 + 2dudv - dy^2 - dz^2\,, \tag{14}$$

More suitable are polar coordinates

$$ds^2 = H(u, \rho, \varphi)du^2 + 2dudv - d\rho^2 - \rho^2 d\varphi^2\,. \tag{15}$$

The solution for H is

$$H = Re\left(\sum_{n=0}^{\infty}\left(A_n(u)J_n(-im\rho)e^{-in\varphi} + B_n(u)Y_n(-im\rho)e^{-in\varphi}\right)\right) \tag{16}$$

Velocity memory effect $\frac{1}{2}H = e^{-(u-10)^2} Re(Y_2(-i\rho)e^{-2i\varphi})$

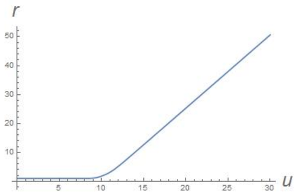

Figure: Graphics r[u], Initial conditions $r[0] = 1$, $r'[0] = 0$, $\varphi[0] = 0$, $\varphi'[0] = 0$.

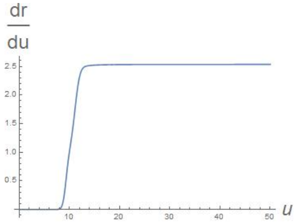

Figure: Graphics dr[u]/du, Initial conditions $r[0] = 1$, $r'[0] = 0$, $\varphi[0] = 0$, $\varphi'[0] = 0$.

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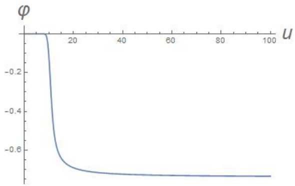

Figure: Graphics φ[u], Initial conditions $r[0] = 1$, $r'[0] = 0$, $\varphi[0] = 0$, $\varphi'[0] = 0$.

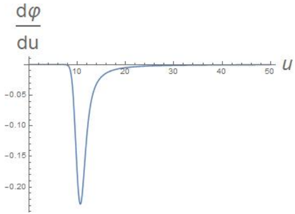

Figure: Graphics dφ[u]/du, Initial conditions $r[0] = 1$, $r'[0] = 0$, $\varphi[0] = 0$, $\varphi'[0] = 0$.

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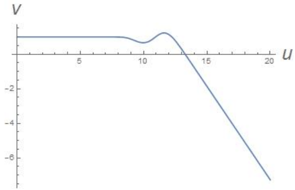

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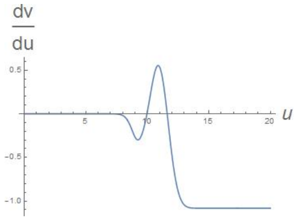

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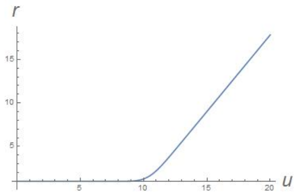

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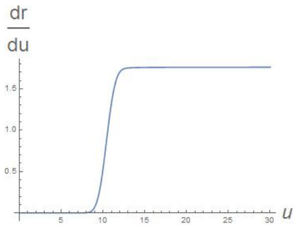

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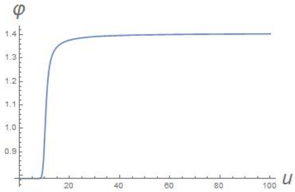

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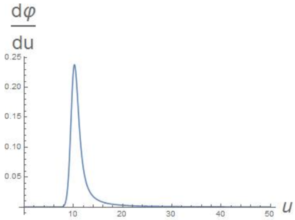

Figure: Graphics dφ[u]/du, Initial conditions $r[0] = 1$, $r'[0] = 0$, $\varphi[0] = \pi/4$, $\varphi'[0] = 0$.

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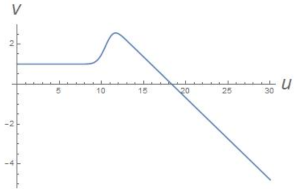

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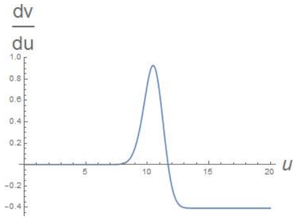

Figure: Graphics dv[u]/du, Initial conditions $v[0] = 1$, $v'[0] = 0$.

Conclusion

- Velocity memory effect in 3D
- Velocity memory effect for massive torsion plane waves
- Soft particles not relevant for memory effect
- Detection of memory effect expected in "closer" future. Maybe possible (indirect) detection of non-zero torsion.

Thank you for your attention

Questions (if any)?